Static Termination Analysis for Event-driven Distributed Algorithms

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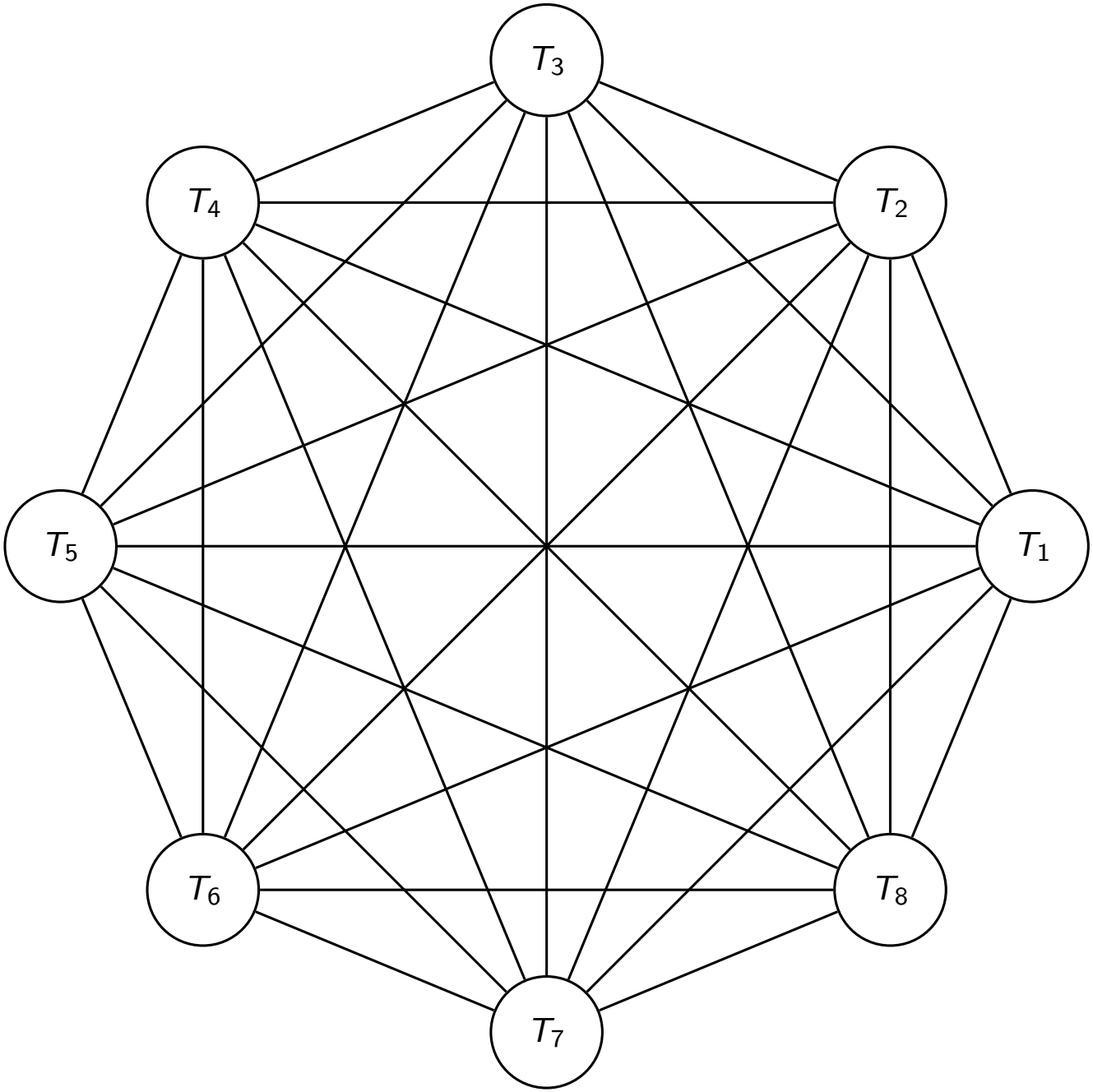
Overview

- **Termination**
  - Important non-functional property
  - Undecidable
- **Approach**
  - Event-based model
  - Criterion implying termination

Model



message-based

point-to-point

asynchronous

Model: Tasks

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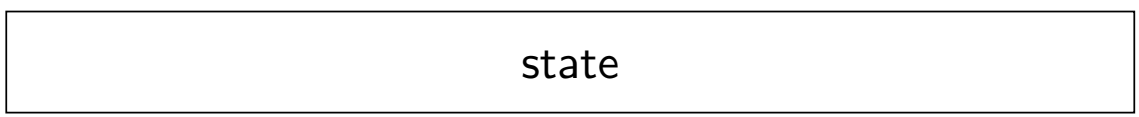

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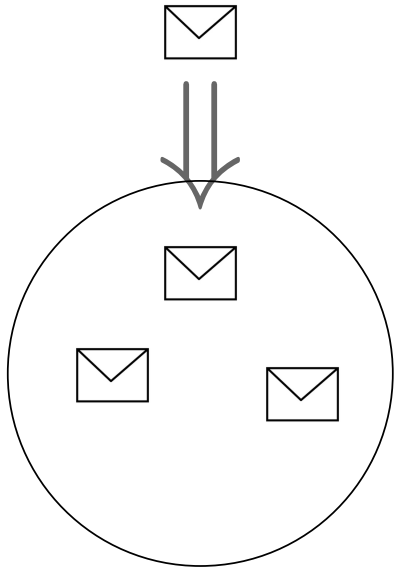
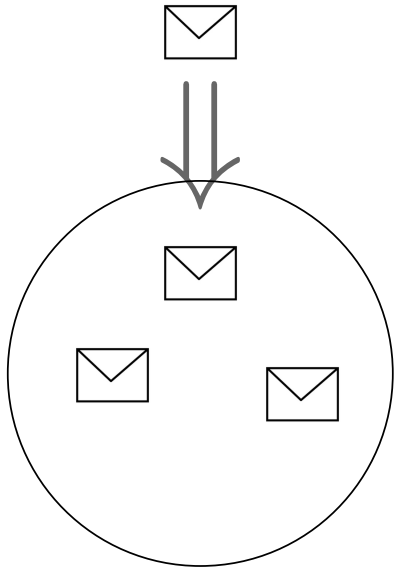
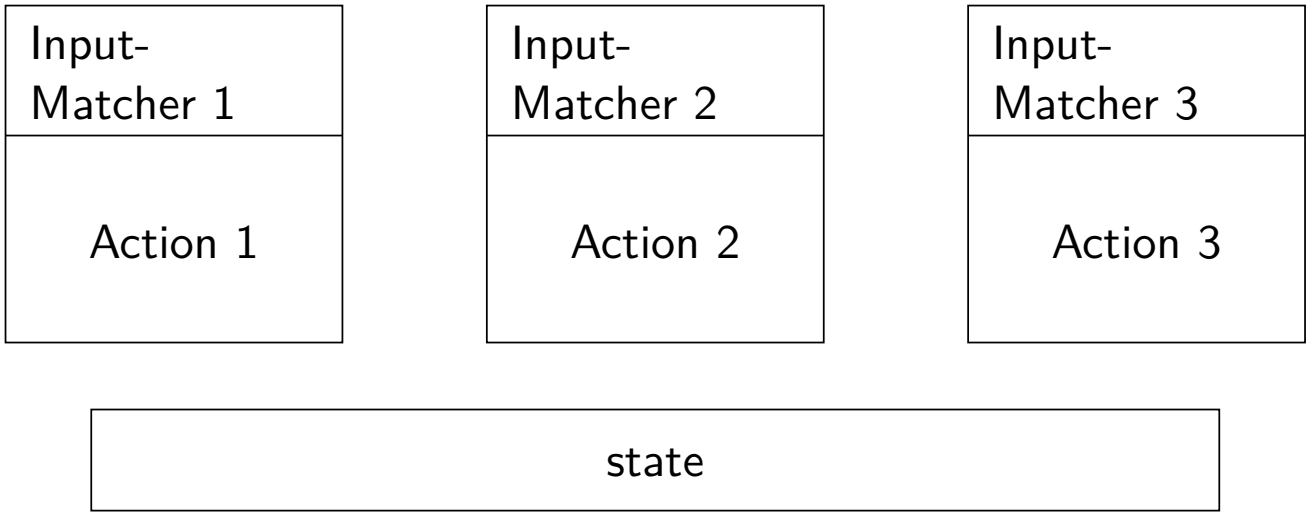


state

Model: Tasks




Input-
Matcher 1
Action 1
Input-
Matcher 2
Action 2
Input-
Matcher 3
Action 3
state

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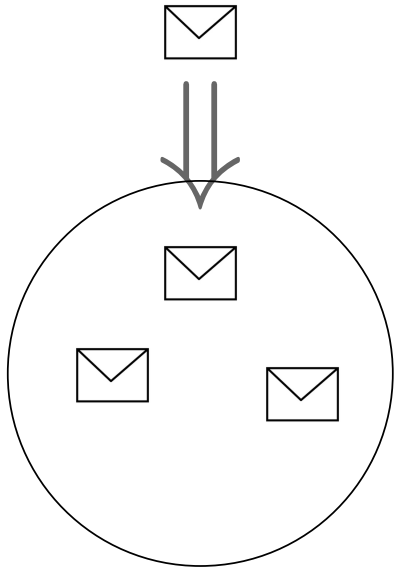
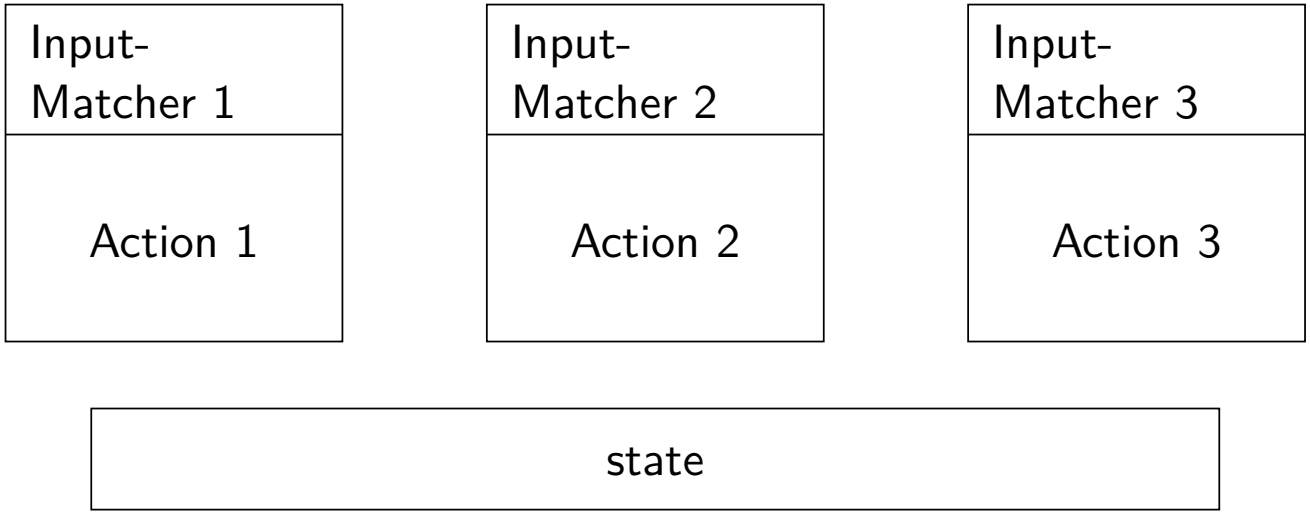


Guarded commands
(Dijkstra 1975)



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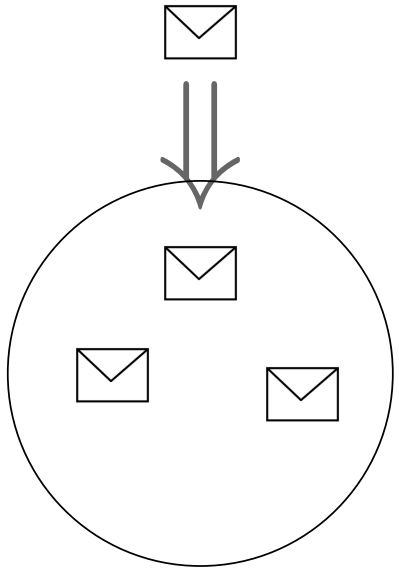

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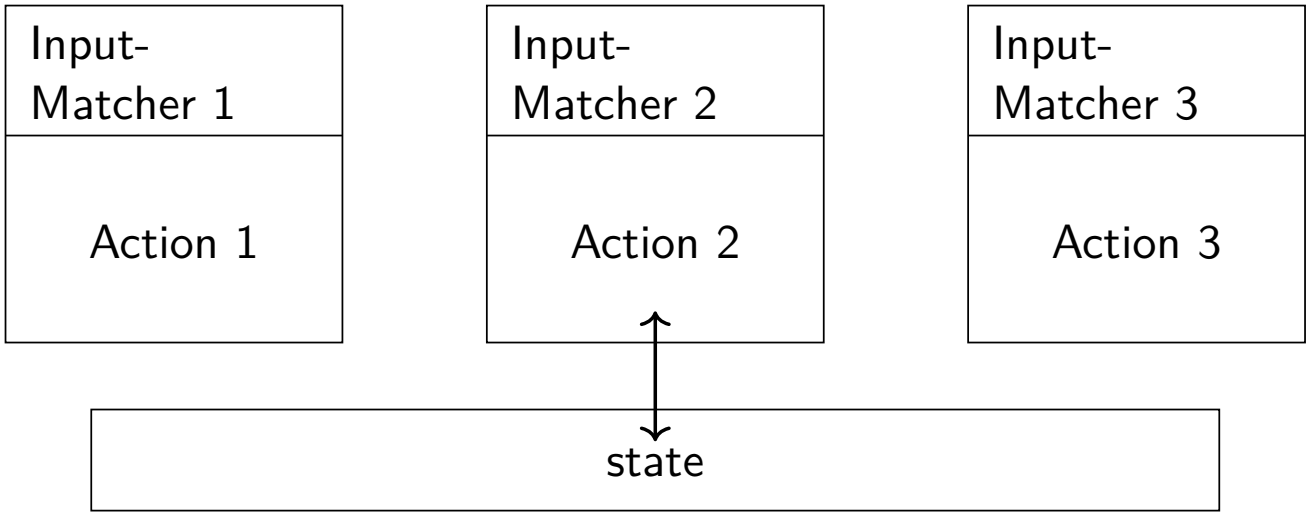

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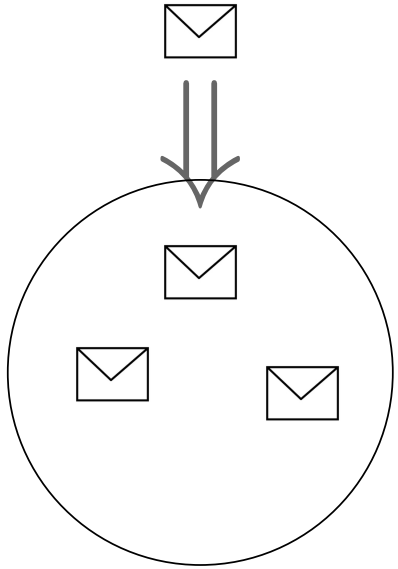

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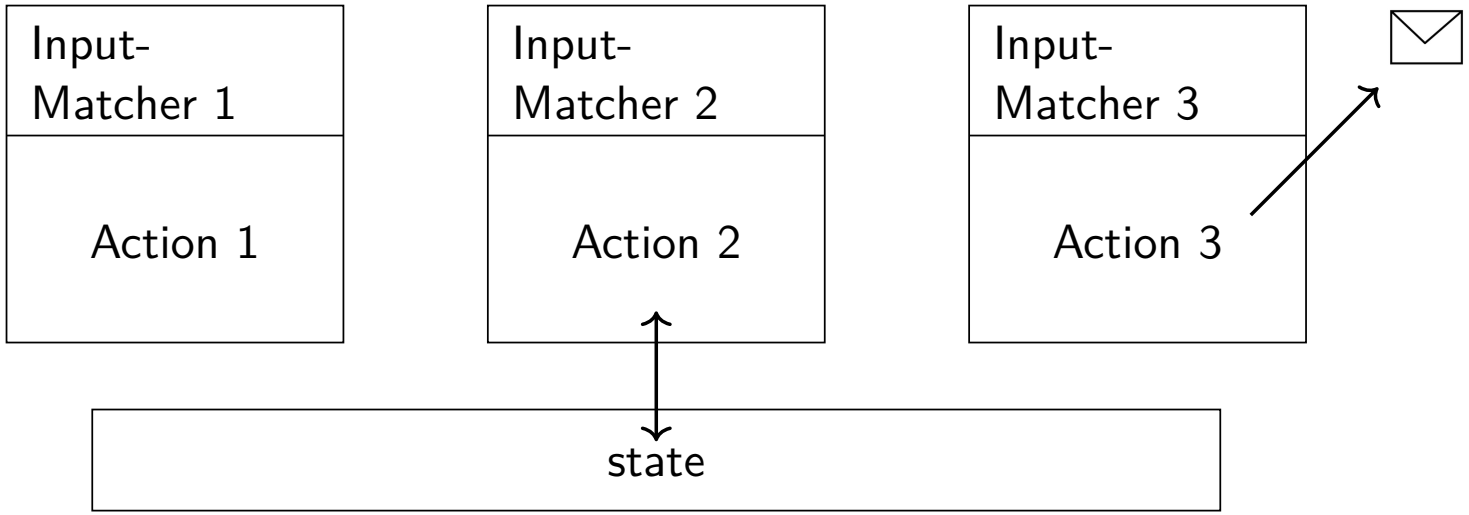

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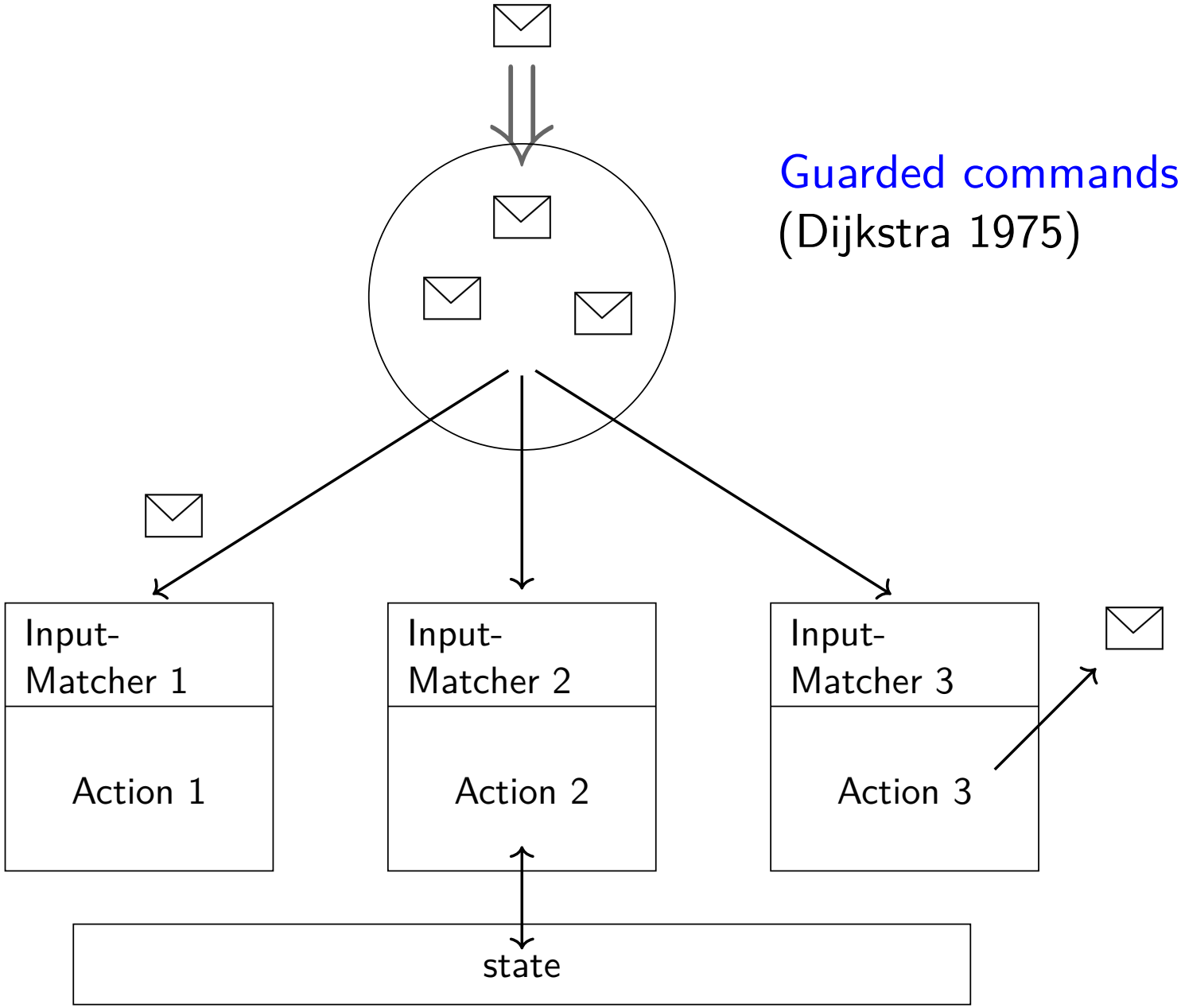

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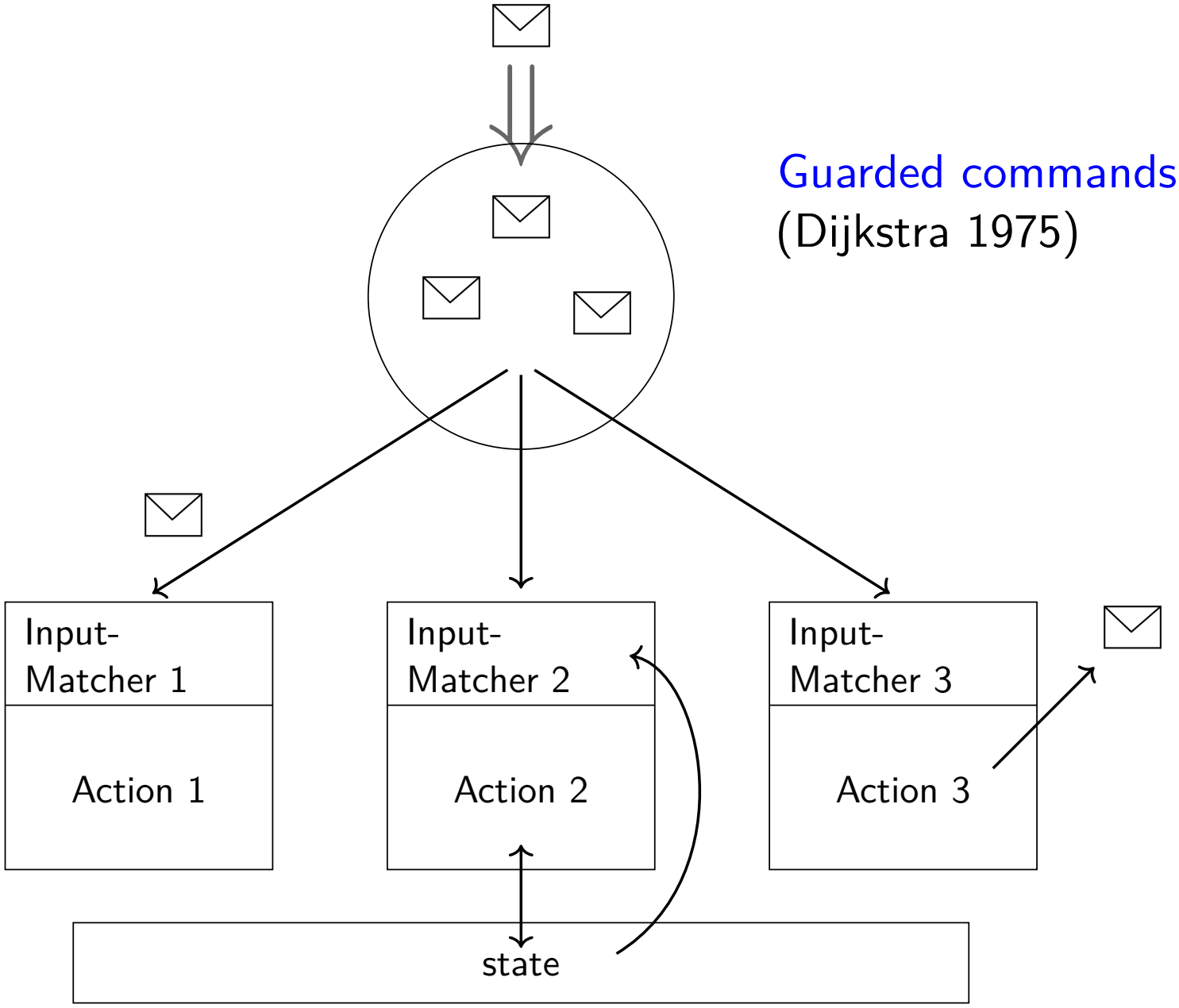

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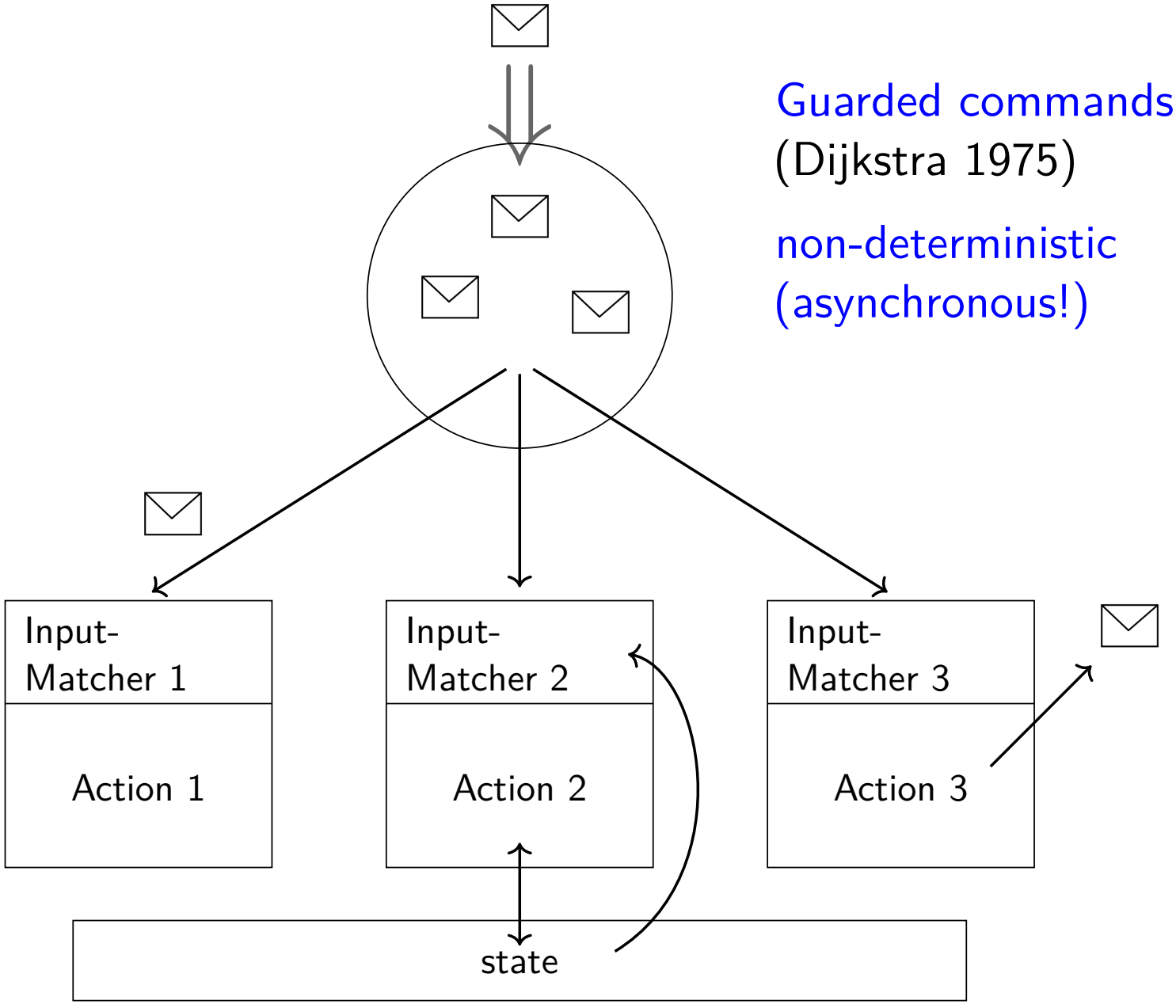

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non-deterministic
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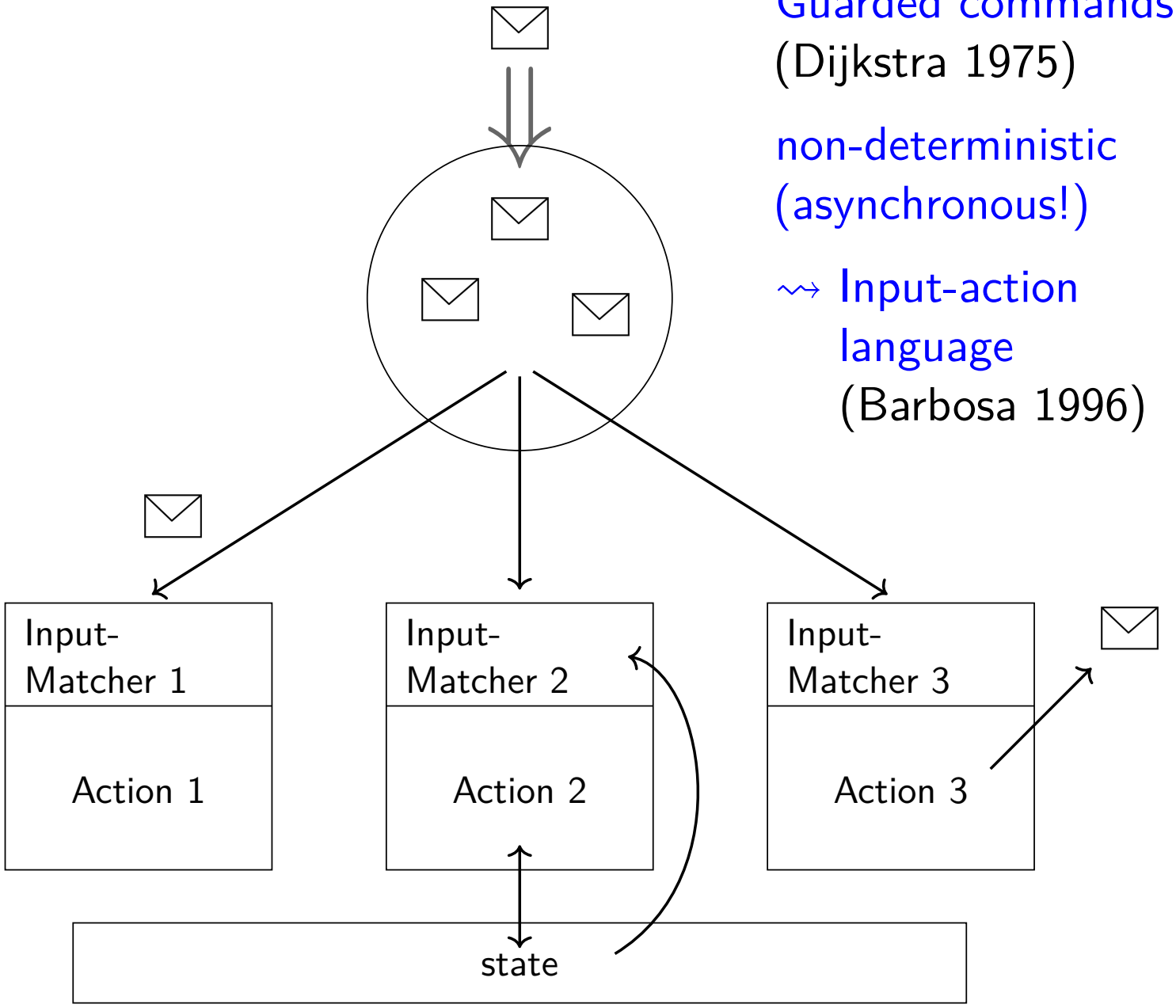

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non-deterministic
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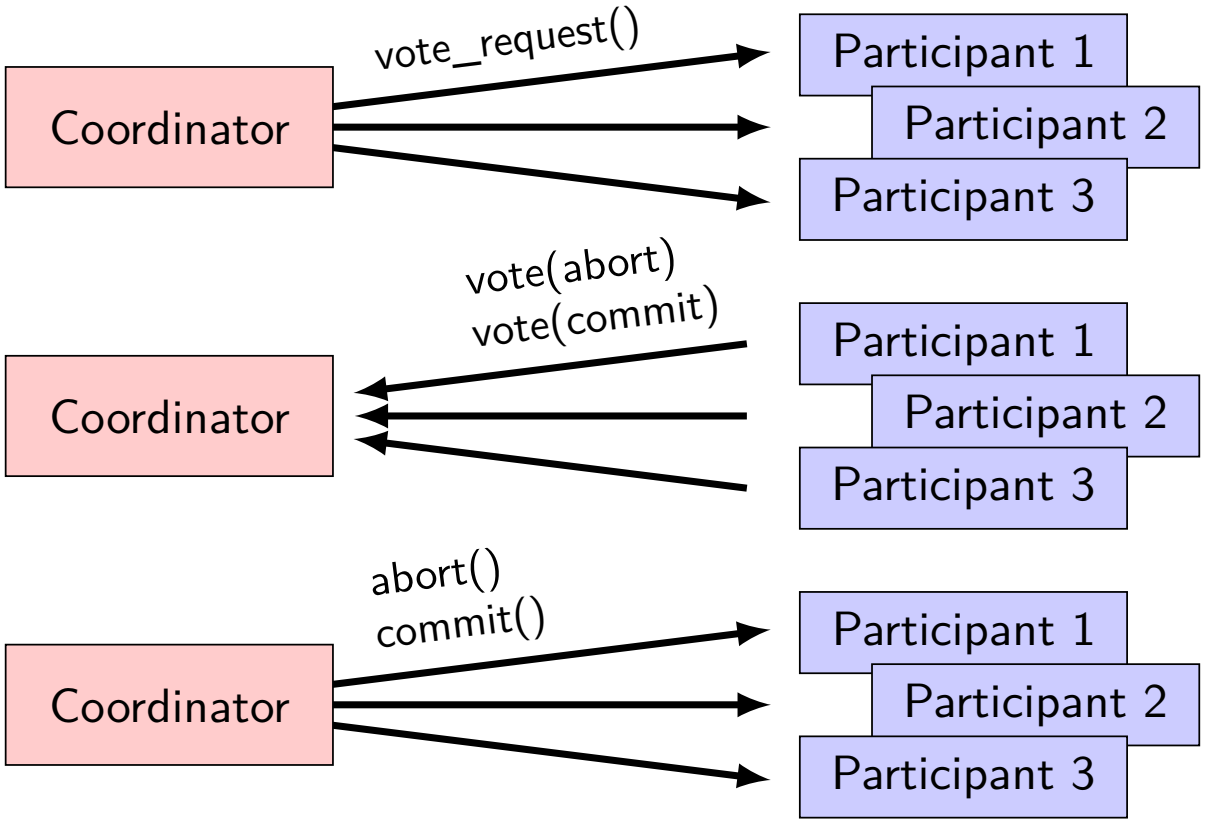
$\rightsquigarrow$ Input-action
language
(Barbosa 1996)

Two-phase commit protocol



Two-phase commit protocol

Participant: $P = \{ P_1, \ldots, P_n \}$

Variables

t // *local part of transaction*
v // *own vote (commit/abort)*

input *vote_request()* {
  reply vote(v)
}

input *abort()* {
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}

input *commit()* {
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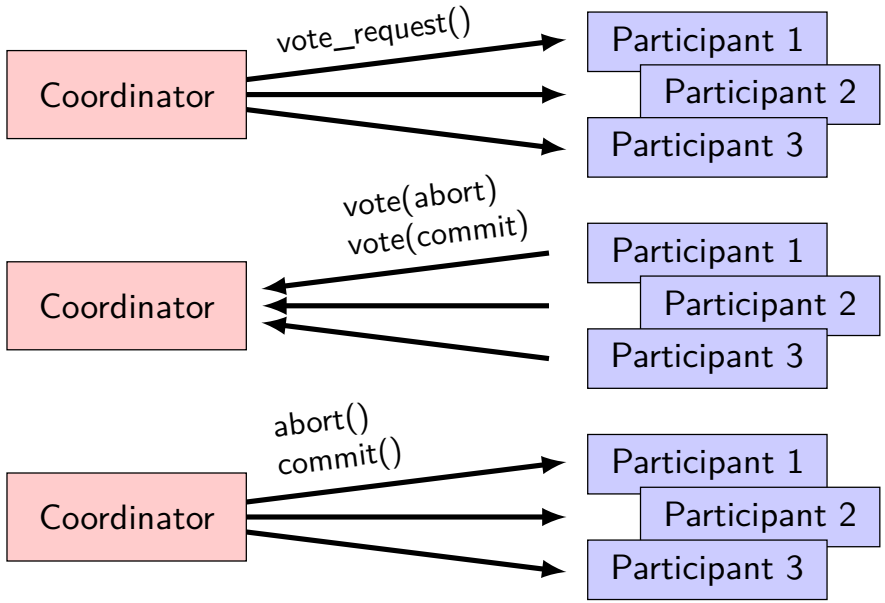

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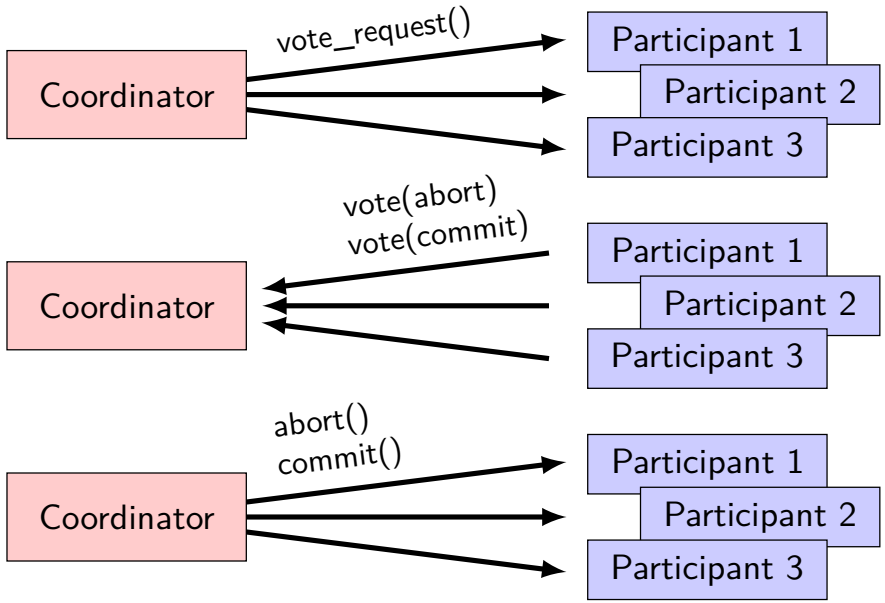

Two-phase commit protocol

Coordinator: { C }

Variables

t // *local part of transaction*
v // *own vote (commit/abort)*
count := 0 // *commit count*

input *init()* {
  if v = abort **then**
    send *abort()* **to** *P*
    t.abort()
  else
    send *vote_request()* **to** *P*
  end
}

input *vote(x)* {
  if x = abort **then**
    send *abort()* **to** *P*
    t.abort()
  else
    count++
    if count = n **then**
      send *commit()* **to** *P*
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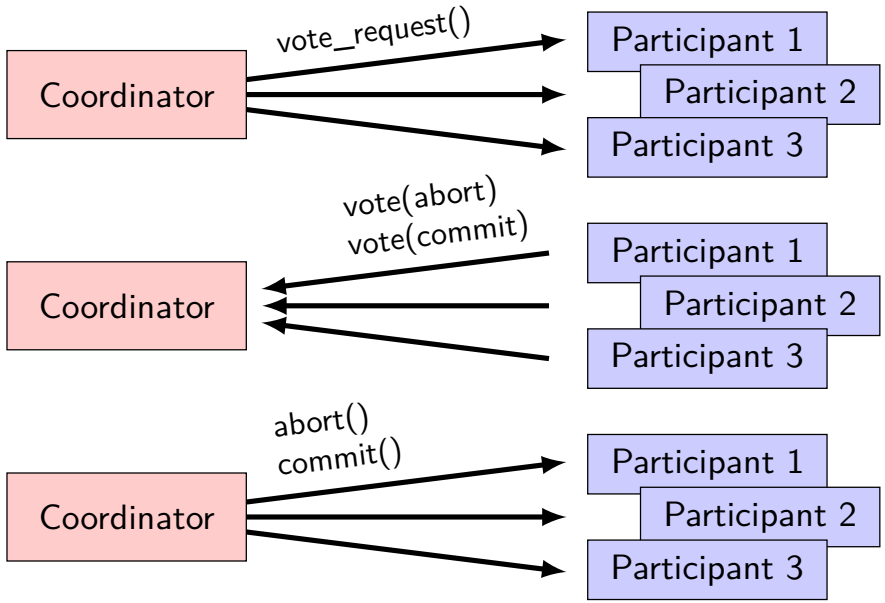

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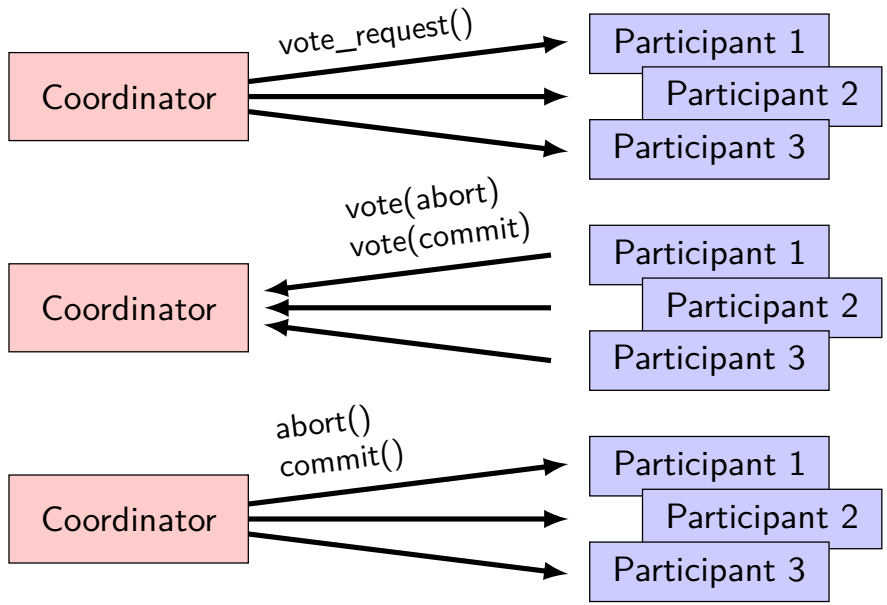

Message flow in the two-phase commit protocol

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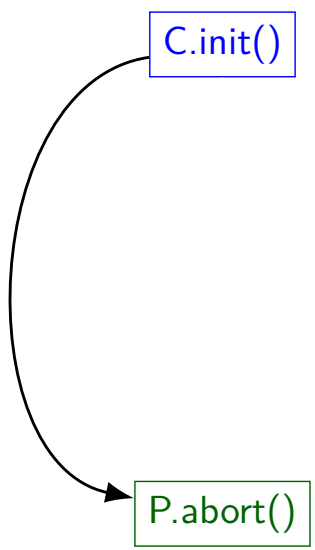

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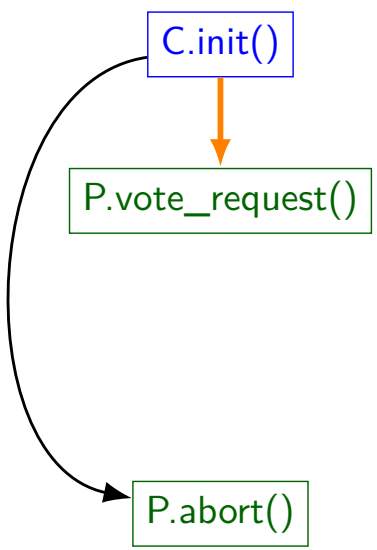

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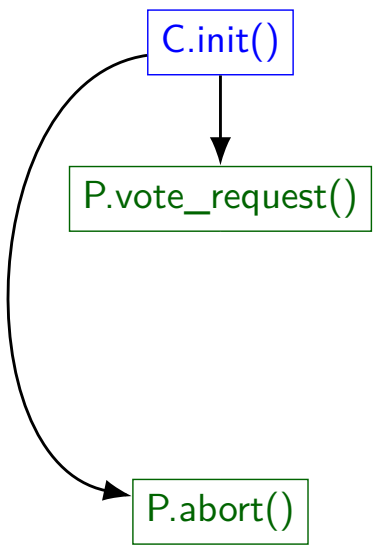

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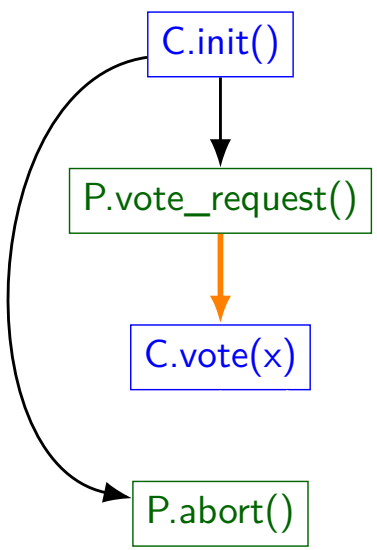

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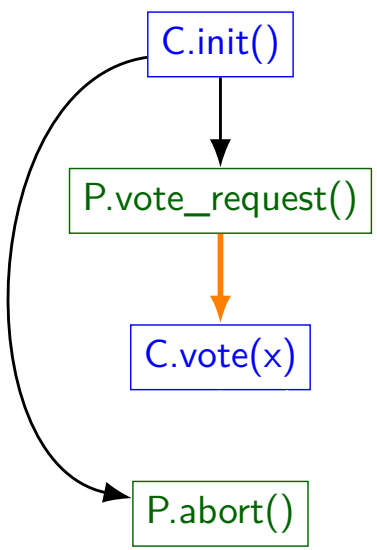

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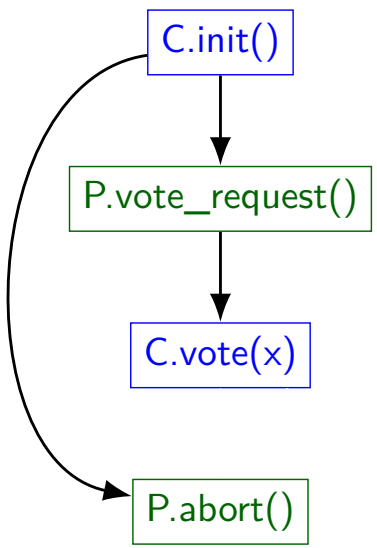

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        t.abort()
    else
        send vote_request() to P
    end
}

input vote(x) {
    if x = abort then
        send abort() to P
        t.abort()
    else
        count++
        if count = n then
            send commit() to P
            t.commit()
        end
    end
}
```

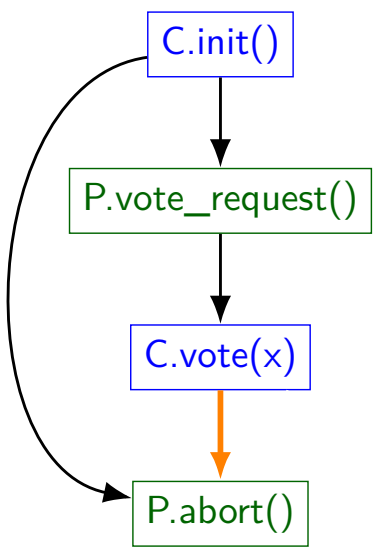
Participant: $P = \{ P_1, \ldots, P_n \}$

```
Variables
    t // local part of transaction
    v // own vote (commit/abort)

input vote_request() {
    reply vote(v)
}

input abort() {
    t.abort()
}

input commit() {
    t.commit()
}
```



Message flow in the two-phase commit protocol

Coordinator: $\{ C \}$

```
Variables
    t // local part of transaction
    v // own vote (commit/abort)
    count := 0 // commit count

input init() {
    if v = abort then
        send abort() to P
        t.abort()
    else
        send vote_request() to P
    end
}

input vote(x) {
    if x = abort then
        send abort() to P
        t.abort()
    else
        count++
        if count = n then
            send commit() to P
            t.commit()
        end
    end
}
```

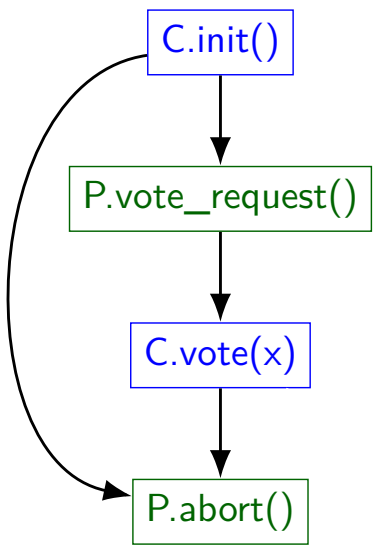
Participant: $P = \{ P_1, \ldots, P_n \}$

```
Variables
    t // local part of transaction
    v // own vote (commit/abort)

input vote_request() {
    reply vote(v)
}

input abort() {
    t.abort()
}

input commit() {
    t.commit()
}
```



Message flow in the two-phase commit protocol

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```
Variables
    t // local part of transaction
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input init() {
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        send abort() to P
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    else
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    end
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        send abort() to P
        t.abort()
    else
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            t.commit()
        end
    end
}
```

Participant: $P = \{ P_1, \ldots, P_n \}$

```
Variables
    t // local part of transaction
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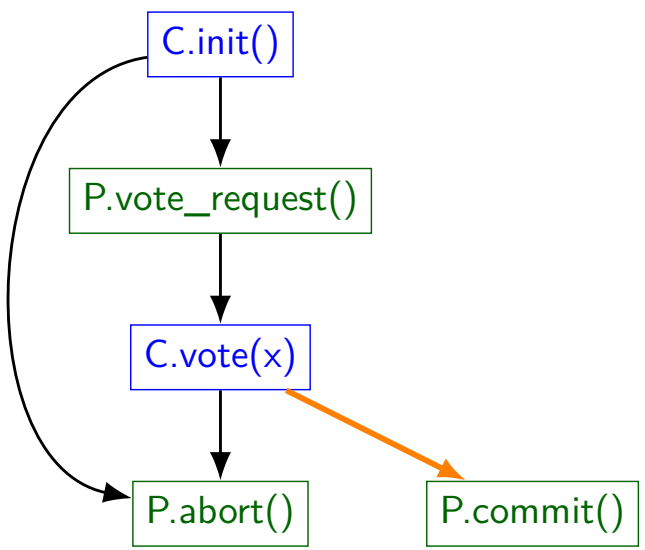
input vote_request() {
    reply vote(v)
}

input abort() {
    t.abort()
}

input commit() {
    t.commit()
}
```



Termination criterion

Theorem

If an algorithm's message flow graph is acyclic, then the algorithm always terminates.

Basic assumptions

- All actions terminate
- No spontaneous actions
  - Each action consumes a message

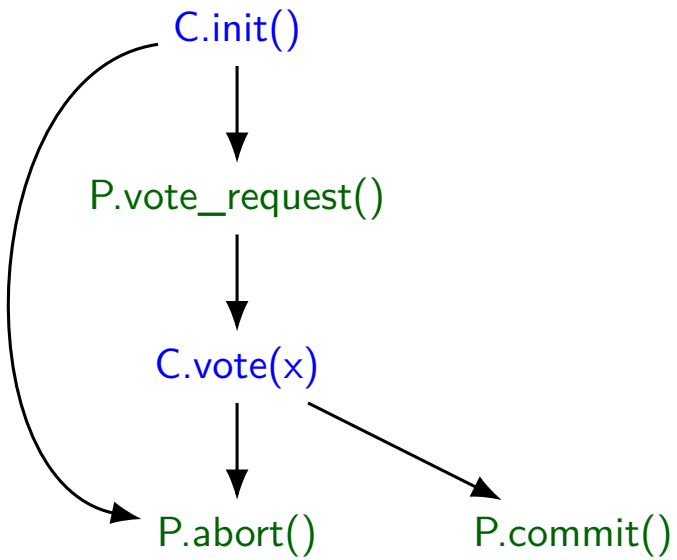
Termination criterion

Theorem

If an algorithm's message flow graph is acyclic, then the algorithm always terminates.

Advantages

- Practical language
- Syntactic criterion
  - Static analysis
  - Efficient: $\mathcal{O}(L + \#IAP^2)$
- Visualization as a tool



Termination criterion

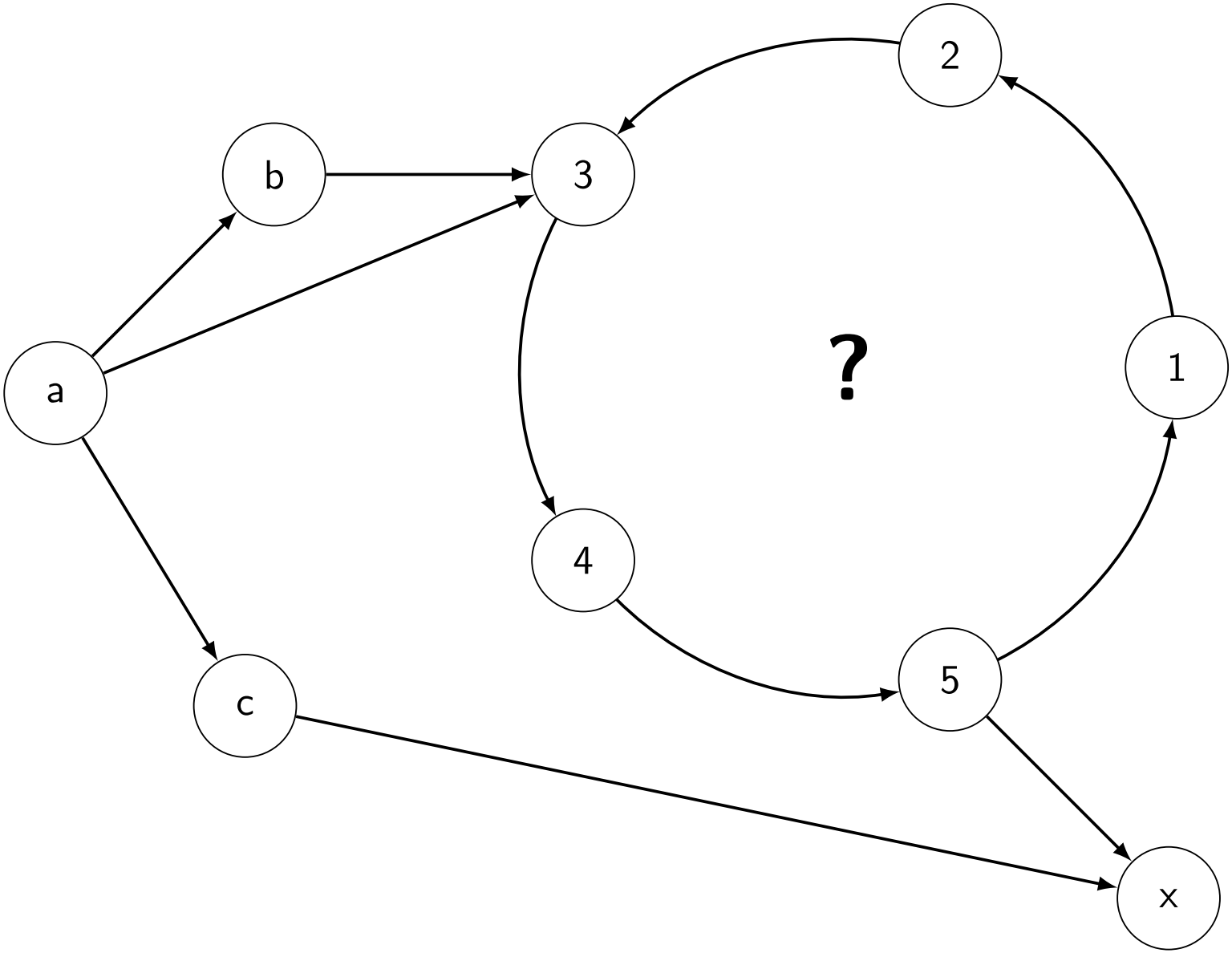
Theorem

If an algorithm's message flow graph is acyclic, then the algorithm always terminates.

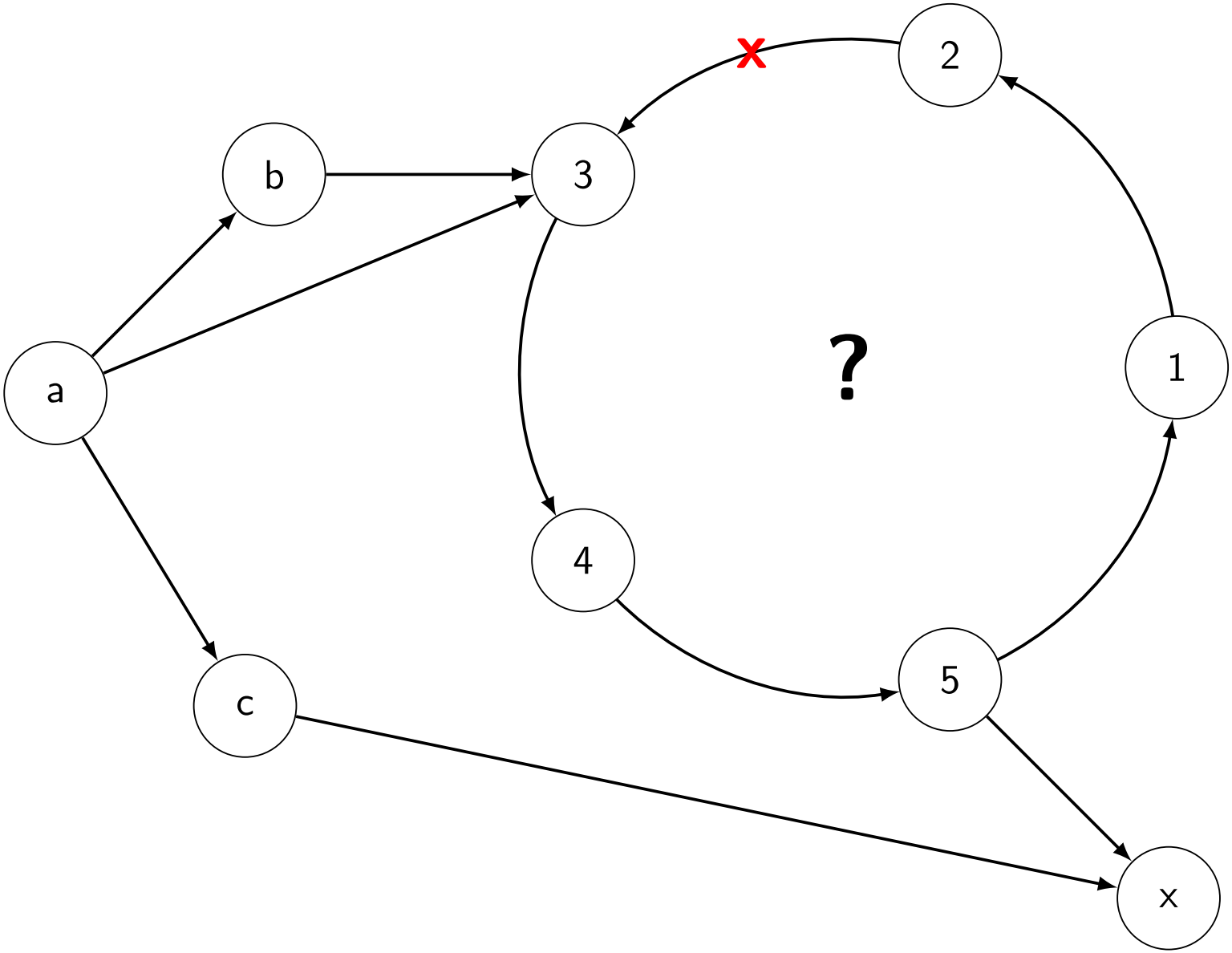
Disadvantages

- No spontaneous actions – timers?
- Precision?
  - Sequential protocols ✓
  - More complicated protocols?

Improving precision

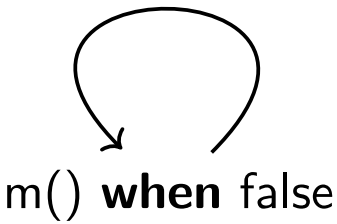


Non-traversable cycles

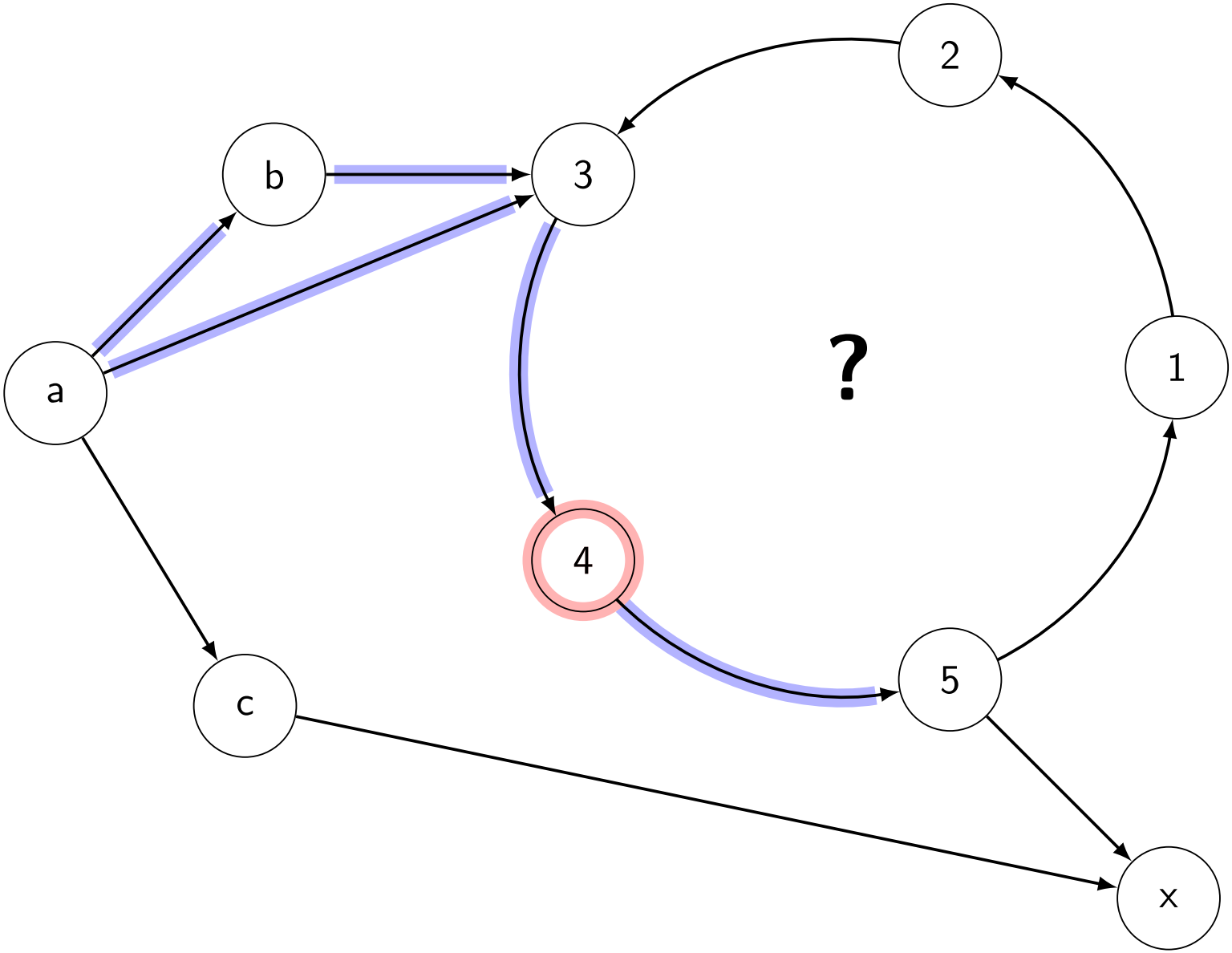


Non-traversable cycles: Impossible message flow

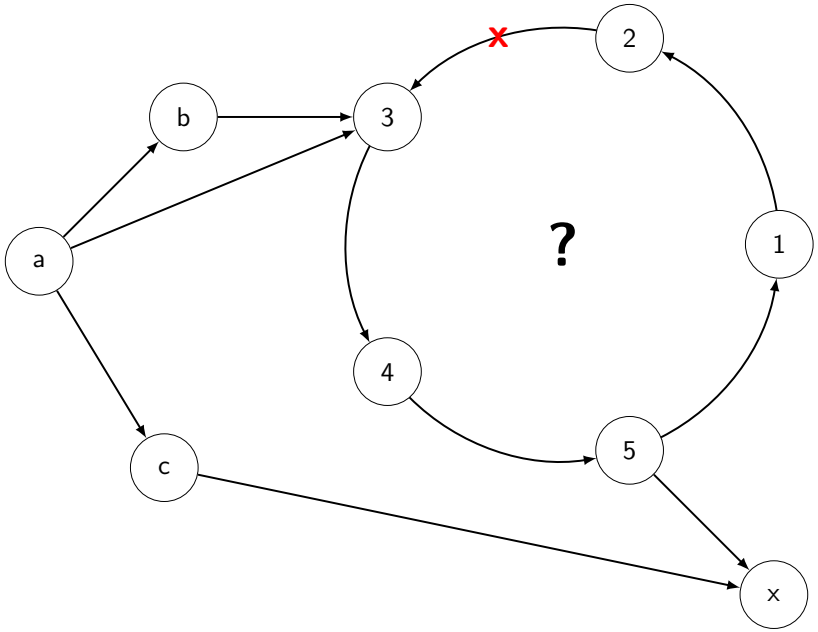
```
T
input m() when false {
    send m() to T
}
```

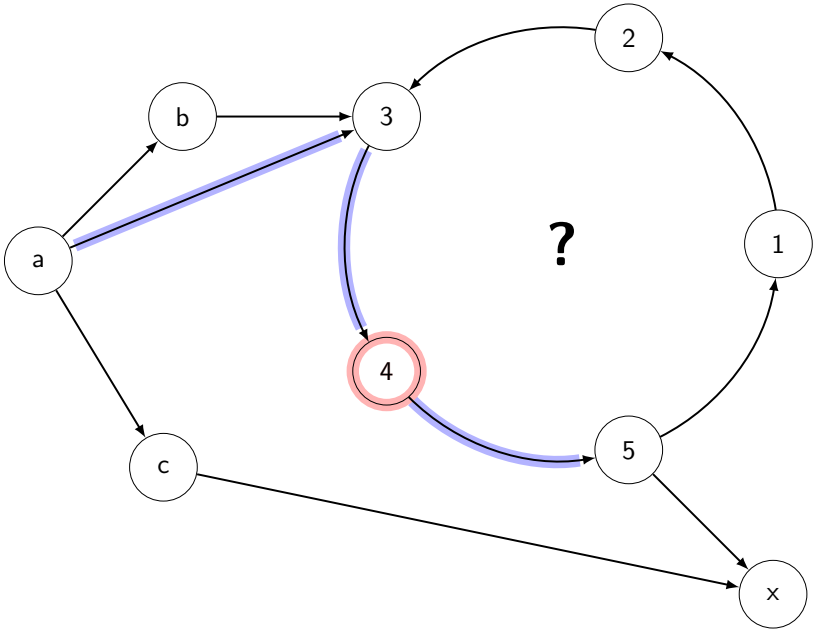


Non-traversable cycles (2)

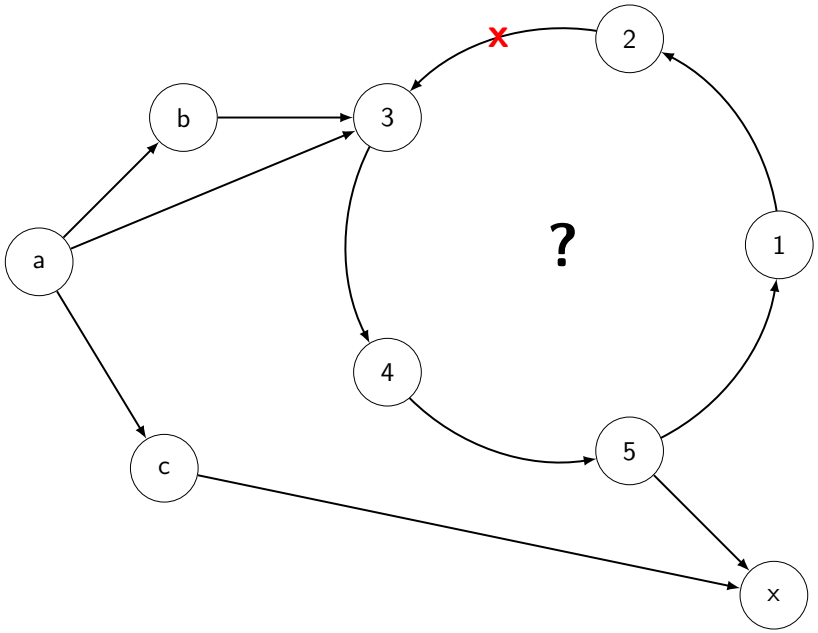


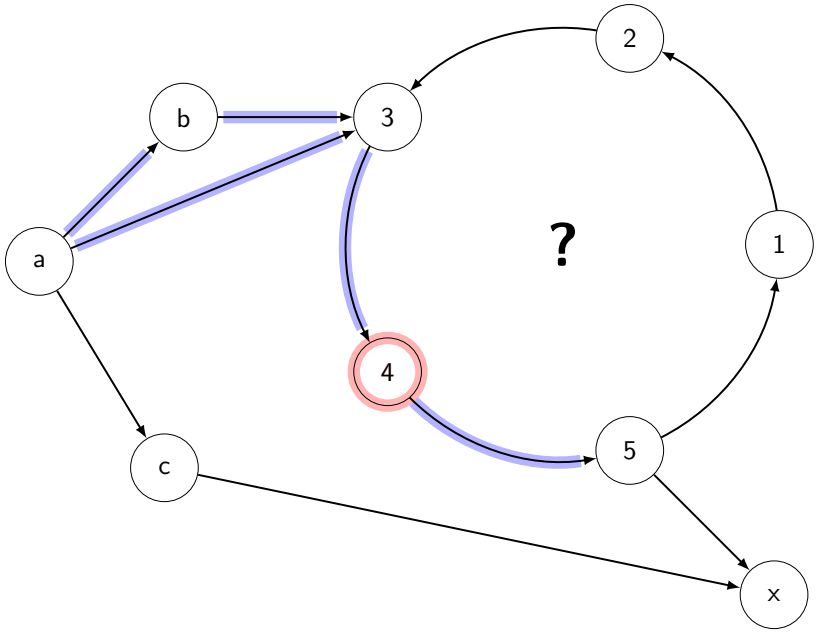
Non-traversable cycles

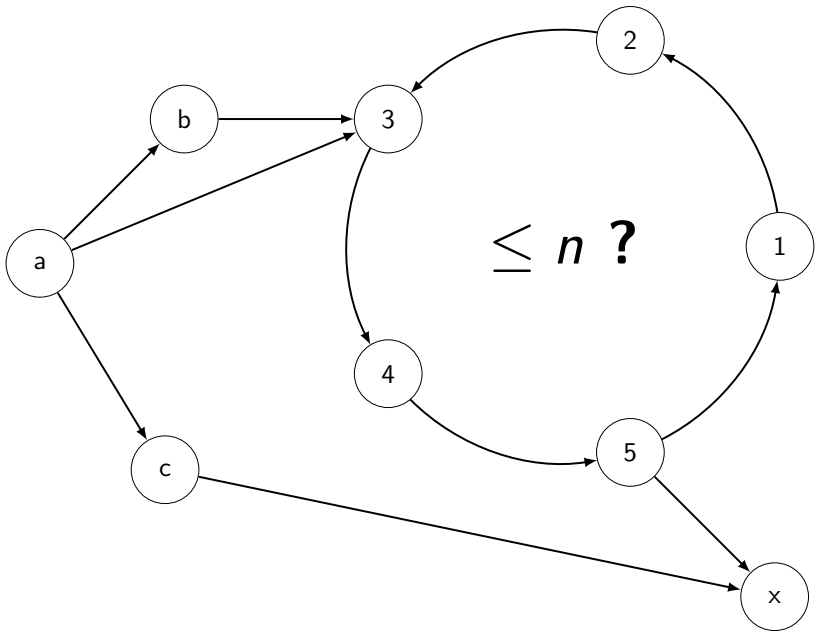




Limited cycles







Limited cycles

Leader election on a ring (Chang/Roberts 1979)

Ring node

Variables

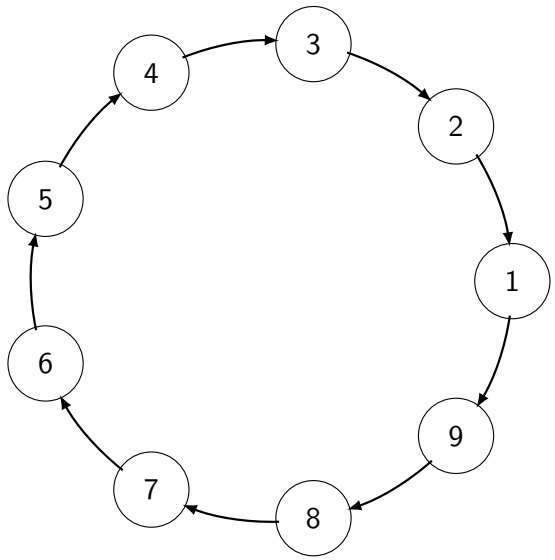
leader // *the current leader*

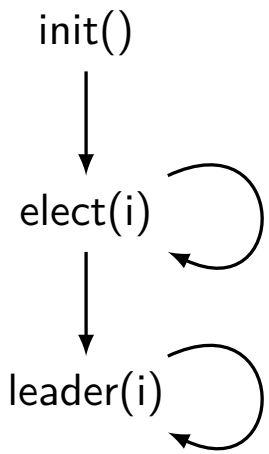
self, next // *own/next ID on ring*

```
input init() {
    send elect(self) to next
}

input elect(i) limit 2 {
    if i = self then
        send leader(i) to next
    else
        send elect(max(i, self)) to next
    end
}

input leader(i) limit 1 {
    leader := i
    if i ≠ self then
        send leader(i) to next
    end
}
```





Chang and Roberts ring algorithm – unrolled

Ring node

Variables
leader, self, next

input init() {
  send elect_1(self) **to** next
}

input elect_1(i) {
  if i = self **then**
    send leader_1(i) **to** next
  else
    send elect_2(max(i, self)) **to** next
  end
}

input elect_2(i) {
  if i = self **then**
    send leader_1(i) **to** next
  else
    send elect_3(max(i, self)) **to** next
  end
}

...

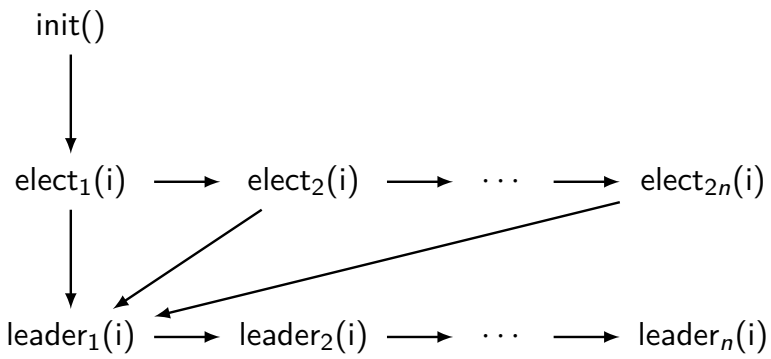
input elect_{2n}(i) {
  if i = self **then**
    send leader_1(i) **to** next
  else
    error "Limit exceeded"
  end
}

input leader_1(i) {
  leader := i
  if i ≠ self **then**
    send leader_2(i) **to** next
  end
}

input leader_2(i) {
  leader := i
  if i ≠ self **then**
    send leader_3(i) **to** next
  end
}

...

input leader_n(i) {
  leader := i
  if i ≠ self **then**
    error "Limit exceeded"
  end
}



Summary

- **Goal:** Static termination analysis for distributed algorithms
- **Approach:**
  - Event-driven model
  - Analyze possible communication between input-action pairs
    $\rightsquigarrow$ Message flow graph
- **Result:** Syntactic termination criterion
  - Acyclicity implies termination
- Improving precision
- **Conclusion:** Framework for static termination analysis
- Implementation: https://github.com/felixwiemuth/JIAL